Independent of Path:

The line integral $\int_C Pdx+Qdy+Rdz$ is independent of the path C in the domain D if & only if $Pdx+Qdy+Rdz$ is exact in D.

If there exists a function $\phi(x,y,z)$ such that $d\phi = Pdx+Qdy+Rdz$

$$\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz = Pdx+Qdy+Rdz$$

$$\Rightarrow \frac{\partial \phi}{\partial x} = P, \quad \frac{\partial \phi}{\partial y} = Q, \quad \frac{\partial \phi}{\partial z} = R,$$

Further, $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}, \quad \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z}$

1. Evaluate $\int_C (4xy-3x^2z^2)dx + 2x^2dy - 2x^3zdz$ along $(3,-1,1)$ to $(0,1,2)$

Sol^n $P = 4xy-3x^2z^2, \quad Q = 2x^2, \quad R = -2x^3z$

$$\frac{\partial P}{\partial y} = 4x, \quad \frac{\partial Q}{\partial x} = 4x, \quad \frac{\partial Q}{\partial z} = 0, \quad \frac{\partial R}{\partial y} = 0$$

$$\frac{\partial R}{\partial x} = -6x^2z, \quad \frac{\partial P}{\partial z} = -6x^2z$$

Here, $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}, \quad \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z}$

Thus $Pdx+Qdy+Rdz$ is exact & hence the given integral is independent of ~~the~~ path

$$Pdx+Qdy+Rdz = (4xy-3x^2z^2)dx + 2x^2dy - 2x^3zdz$$

$$= 4xydx + 2x^2dy - 3x^2z^2dx - 2x^3zdz$$

$$= d(2x^2y) - d(x^3z^2)$$

$$\therefore \int_C Pdx+Qdy+Rdz = \int_{(3,-1,1)}^{(0,1,2)} d(2x^2y) - d(x^3z^2)$$

$$= \left(2x^2y - x^3z^2\right)_{(3,-1,1)}^{(0,1,2)} = 45$$

Problems

1) S.T $\int_C (y^2dx + 2xydy)$ is independent of the path joining $(0,1)$ & $(1,3)$ & hence evaluate.

2) Evaluate $\int_C 2xydx + (x^2+2yz)dy + (y^2+1)dz$ along 'c' from $(0,0,0)$ to $(1,1,1)$

3) Show that $\int_C (2xyz^2)dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz$ along the curve C from $(0,0,1)$ to $(1, \frac{\pi}{2}, 2)$ is $\pi+1$.